36<sup>th</sup> International Electric Vehicle Symposium and Exhibition (EVS36) Sacramento, California, USA, June 11-14, 2023

# Modified Pure Pursuit Algorithm Robust to Localization Noise

Kyung-Ho Kim<sup>1</sup>, Hoyong Na<sup>1</sup>, Jihyeok Ahn<sup>1</sup>, Sujin Shin<sup>1</sup>, Sung-Ho Hwang<sup>1</sup>

<sup>1</sup>Department of Mechanical Engineering, Sungkyunkwan University, Republic of Korea, hsh0818@skku.edu

#### **Executive Summary**

The localization process is crucial for the proper functioning of autonomous vehicles, and any noise introduced during this process can have negative effects on the vehicle's safety and ride quality. Specifically, noise generated in the localization process tends to be transmitted to the steering control, thereby interfering with desirable control. In this study, a modified pure pursuit algorithm was developed to prevent such noise from adversely affecting steering control. The effectiveness of the proposed algorithm was validated through path-tracking simulations, which demonstrated that it was successful in significantly reducing the vibration of steering and the magnitude of the lateral jerk, both under straight and curved paths.

Keywords: Autonomous vehicle, Control system, GPS, Simulation, Strategy

### **1** Introduction

The current autonomous driving localization algorithm is performed mainly by using GNSS (Global Navigation Satellite System) and inertial sensors [1]. To increase accuracy, various signal processing and sensor fusion algorithms are being studied. In [2], localization accuracy was enhanced by integrating map information with the GNSS and inertial sensor measurements. A method for enhancing the performance of EKF-based GNSS and IMU fusion has been proposed, which integrates pseudorange error predictions to improve localization accuracy [3]. Alternatively, the use of LiDAR to obtain additional environmental information for positioning has also been studied [4,5]. Nevertheless, it is impossible to completely remove noises arising from various factors. If such noises are left unattended, there is a problem of degrading the performance of the subsequent autonomous driving algorithms and eventually causing the vehicle to be unstably controlled. This is because most autonomous driving lateral control algorithms are based on the geometric relationship between the current location of the vehicle and the path to be followed. However, if a time-series filter is used to remove noise, a delay occurs in the signal, causing another problem the vehicle deviates from the intended path.

To address the aforementioned issue, it is imperative to utilize a lateral control algorithm that effectively mitigates the transmission of noise or conduct a post-processing procedure to eliminate any noise present in the input signal from the existing controller. This study proposes a steering control regulation to improve the problem of control instability due to noise generated during the localization process. Considering the level of a positioning error, it could refrain from imposing excessive restrictions on steer input, simultaneously regulating the occurrence of chattering steering motion, typically caused by localization noise, to minimize unnecessary lateral jerk.

### 2 Modified Pure Pursuit Algorithm

#### 2.1 Basic Pure Pursuit Algorithm

The pure pursuit algorithm sets the look-ahead point expected for the vehicle to reach within the path and derives a steering angle that lets the vehicle reach that point by forming a circular trajectory [6]. In general, the distance to the look-ahead point is proportional to the vehicle's speed and inversely proportional to the curvature of the path [7]. The equation for calculating the steering angle is as follows, in which R,  $\alpha$ , L,  $\delta$ , and  $l_d$  represent the radius of curvature, heading difference between the look-ahead point and the ego vehicle, wheelbase, steering angle, and look-ahead distance, respectively.

$$2R \cdot \sin \alpha = L \tag{1}$$
$$\delta = \operatorname{atan}\left(\frac{2L \sin \alpha}{l_d}\right) \tag{2}$$

The pure pursuit method has the advantage of requiring the adjustment of only one parameter, the look-ahead distance. The larger the value, the better the lateral stability of the vehicle, but at the cost of slower convergence speed of the path error. In contrast, if the look-ahead distance value is set too small, the steering angle may vibrate or diverge due to disturbances, especially when noise is generated during the localization process. In practice, the look-ahead distance is tuned to a slightly larger value to prevent this problem when using the pure pursuit algorithm.

### 2.2 Formulation of the Modified Pure Pursuit Algorithm

In this section, we will present the features and implementation methods of the proposed algorithm. The primary contribution of the proposed method lies in its ability to exhibit a small computational burden and high adaptability. The algorithm involves straightforward calculations that can be executed with limited memory, ensuring real-time performance in a real vehicle environment. Furthermore, the proposed algorithm can be universally applied to the rear of various steering control algorithms without compromising their performance to a significant level. This paper presents an example of the proposed algorithm applied to the pure pursuit algorithm. However, it should be noted that the algorithm can be applied based on the steering angle and positioning results that any other steering controllers initially determine.

In the proposed modified pure pursuit algorithm, the look-ahead point can move within an allowable range. As illustrated in Figure 1, the algorithm first determines the look-ahead line based on the predetermined look-ahead distance and estimated localization error range. Subsequently, the final look-ahead point is chosen from the look-ahead line.



Figure1: Definition of look-ahead line

#### 2.2.1 Determination of look-ahead line

The location of the center point of the look-ahead line, which would be denoted as  $X_C(x_C,y_C)$ , is on the pursuing path as far as the look-ahead distance from the center of the rear wheel of the ego vehicle. Thereafter, the look-ahead line is completed by extending the line perpendicular to the pursuing path from the chosen center point by a predetermined length. The left and right endpoints of the look-ahead line will be expressed as  $X_L(x_L,y_L)$ , and  $X_R(x_R,y_R)$ , respectively. The method of determining the length to extend the look-ahead line is as follows. As shown in Fig. 2, localization points measured within the past 1 second are collected. To obtain the component of covariance perpendicular to the vehicle's traveling direction from these localization points, the covariance matrix for the x and y components of the localization points was diagonalized, and the square root of the smaller diagonal element was defined as sigma. Suppose we assume that the lateral localization error follows the normal distribution. In that case, 95% of the localization points may be located inside the band with a thickness of 4 sigmas, as shown in Fig. 2. Epsilon is defined as four times sigma, and it becomes the length of the look-ahead line. Denoting the heading angle of the path at the predetermined look-ahead point  $X_C$  as  $\theta_P$ ,  $X_L$  and  $X_R$  are calculated as follows:

$$X_L = \left(x_C + 2\sigma\cos\left(\theta_P + \frac{\pi}{2}\right), y_C + 2\sigma\sin\left(\theta_P + \frac{\pi}{2}\right)\right) = \left(x_C - 2\sigma\sin(\theta_P), y_C + 2\sigma\cos(\theta_P)\right)$$
(3)

$$X_R = \left(x_C + 2\sigma \cos\left(\theta_P - \frac{\pi}{2}\right), y_C + 2\sigma \sin\left(\theta_P - \frac{\pi}{2}\right)\right) = \left(x_C + 2\sigma \sin(\theta_P), y_C - 2\sigma \cos(\theta_P)\right)$$
(4)



Figure2: Determination of the length of the look-ahead line

#### 2.2.2 Determination of the look-ahead point

To determine the final look-ahead point, it is first necessary to calculate the steering angle required to reach  $X_L$  and  $X_R$ , based on the assumptions made in the pure pursuit algorithm. Let  $l_L$ ,  $l_R$  be the distances from the center of the rear axle of the ego vehicle to  $X_L$  and  $X_R$ , respectively. If  $\delta_L$  and  $\delta_R$  are steering angles to reach  $X_L$  and  $X_R$ , respectively,  $\delta_L$  and  $\delta_R$  can be obtained as follows:

$$\delta_L = \operatorname{atan}\left(\frac{2L\sin\alpha_L}{l_L}\right), \delta_R = \operatorname{atan}\left(\frac{2L\sin\alpha_R}{l_R}\right)$$
(5)

, where  $\alpha_L$  and  $\alpha_R$  are heading differences between the left, and right look-ahead points and the ego vehicle, respectively.

After calculating  $\delta_L$  and  $\delta_R$ , the current steering angle  $\delta_t$  is compared to those values. Note that  $\delta_L$  is always larger than  $\delta_R$  since the counterclockwise steering angle is considered positive. Depending on the size of  $\delta_t$ , the final steering input  $\delta_{t+1}$  is determined as follows:

EVS36 International Electric Vehicle Symposium and Exhibition

$$\delta_{t+1} = \begin{cases} \delta_L & \text{if } \delta_L \le \delta_t \\ \delta_t & \text{if } \delta_R < \delta_t < \delta_L \\ \delta_R & \text{if } \delta_t \le \delta_R \end{cases}$$
(6)

If the current steering angle falls between  $\delta_L$  and  $\delta_R$ , the current steering angle is maintained because the deviance of the steering angle between  $\delta_L$  and  $\delta_R$  is deemed to be due to localization noise. Otherwise, if the steering angle falls outside this range, one of the two values that is closer to the current steering angle is used instead to guarantee the path-tracking performance. As a result, the look-ahead point moves on the look-ahead line while properly reducing the movement of the steering wheel.

### 3 Simulation

To confirm the effect of the proposed look-ahead point-choosing method, the existing pure pursuit algorithm and the proposed algorithm were compared in two scenarios: a straight path and a curved path. A lightweight vehicle model with a wheelbase of 2.97m was utilized for simulation. To simulate the effects of localization noise, Gaussian noise with zero-mean and a standard deviation of 0.6m was employed in the experiment. Path tracking performance, change of steering angle, lateral acceleration, and lateral jerk were compared.

#### 3.1 Straight Path

The straight path scenario was designed to drive 250m of road at constant speed of 50km/h. The look-ahead distance was set to 15m. Comparison of the performance of the pure pursuit algorithm with the proposed algorithm indicated that when the vehicle was controlled by the former, the steering angle experienced vibrations as a result of the localization noise, and the lateral jerk level was observed to be higher than that achieved with the latter.



Figure3: Comparison of motions under a scenario with a straight path

#### 3.2 Curved Path

For the curved path scenario, the vehicle was set to travel along a curved path with a radius of 20m, which included a 270-degree turn. The vehicle was configured to maintain a constant speed of 20km/h, while the look-ahead distance was set to 10m. In this case, the proposed algorithm also yielded a more modest steering angle and reduced lateral jerk, while maintaining lateral acceleration levels that were either similar or lower in magnitude compared to that of the case controlled by the pure pursuit algorithm.



Figure4: Comparison of motions under a scenario with a curved path

### **4** Discussion

The RMS values for path error, steering angular velocity, lateral acceleration, and lateral jerk were compared in Table 1 and Table 2.

	Pure pursuit algorithm	Modified pure pursuit algorithm
RMS path error(m)	1.4038	1.3976
RMS steering angle(deg/s)	39.265	5.1823
RMS lateral acceleration(m/s <sup>2</sup> )	1.0594	0.8284
RMS lateral jerk(m/s <sup>3</sup> )	9.0473	5.3214

Table 1: Comparison of control performances under the straight path scenario

EVS36 International Electric Vehicle Symposium and Exhibition

	Pure pursuit algorithm	Modified pure pursuit algorithm
RMS path error(m)	0.0847	0.1411
RMS steering angle(deg/s)	82.643	13.983
RMS lateral acceleration(m/s <sup>2</sup> )	1.1434	1.1566
RMS lateral jerk(m/s <sup>3</sup> )	9.1170	4.4089

Table 2: Comparison of control performances under the curved path scenario

The lateral acceleration level showed only minor changes under both scenarios, while the deviation from the intended path increased by 0.0564m under the curved path scenario. However, given the high level of localization noise, whose standard deviation is measured at 0.6m, it is difficult to conclude that the vehicle's path-tracking performance has significantly deteriorated. The implementation of the proposed algorithm resulted in a significant modification of the steering angle input signal, as evidenced by the reduced steering angular velocity. Due to the changes above, the lateral jerk was observed to decrease by nearly half.

## 5 Conclusion

To alleviate the problem of steering vibration due to localization noise, the modified pure pursuit algorithm was developed as a lateral control algorithm. This algorithm enables the look-ahead point to move within a specific range that is determined by the localization noise level. The proposed algorithm significantly improved the stability of the steering movement, resulting in enhanced lateral stability and increased passenger comfort.

For future work, we suggest investigating the combination of the modified pure pursuit algorithm with other steering algorithms and improving the steering angle regulation method. In the context of combining the modified pure pursuit algorithm with other steering algorithms, most existing processes, such as determining the localization noise level, remain unchanged. Only the part that computes the steering command to reach the target points, which are shifted to the left and right based on the localization noise, can be modified using different algorithms. Various techniques could be explored to improve the steering angle regulation algorithm, such as utilizing 1-D mapping to adjust the steering angular velocity rate limiter based on the localization noise level instead of merely maintaining the steering angle.

## Acknowledgments

This work was supported in part by the Technology Innovation Program (20014983, Development of autonomous chassis platform for a modular vehicle) funded by the Ministry of Trade, Industry & Energy (MOTIE, Korea), and in part by the Korea Institute for Advancement of Technology grant funded by the MOTIE "The Competency Development Program for Industry specialist" (Foster R&D specialist of parts for eco-friendly vehicle (xEV), under Grant P0017120).

## References

- [1] K. Sampo, et. Al., A Survey of the State-of-the-Art Localization Techniques and Their Potentials for Autonomous Vehicle Applications, IEEE Internet of Things Journal, ISSN 2327-4662, 5(2018), 829-846
- [2] M. M. Atia, S. L. Waslander, *Map-aided adaptive GNSS/IMU sensor fusion scheme for robust urban navigation*, Measurement, ISSN 0263-2241, 131(2019), 615-627
- [3] R. Sun, Z. Zhang, Q. Cheng, et. Al., *Pseudorange error prediction for adaptive tightly coupled GNSS/IMU navigation in urban areas*, GPS Solut, ISSN 1521-1886, 26(2022), 28
- C. Debeunne, D. Vivet, A Review of Visual-LiDAR Fusion based Simultaneous Localization and Mapping, Sensors, EISSN 1424-8220, 20 (2020) 2068

- [5] M. Park, S.H. Hwang, Localization using Dual Fail/Safe Filters with Sensor Fusion in Complex Urban Environments, Presented at the 33rd Electric Vehicle Symposium & Exposition presented by the Electric Drive Transportation Association (EDTA) (EVS33), Zenodo, https://doi.org/10.5281/zenodo.4062844
- [6] R. Craig Coulter, *Implementation of the Pure Pursuit Path Tracking Algorithm*, Carnegie-Mellon UNIV Pittsburgh PA Robotics INST, (1992)
- [7] S. Moveh, M. Hussein, and M. B. Mohamad, A review of some pure-pursuit based path tracking techniques for control of autonomous vehicle, International Journal of Computer Applications, ISSN 0975-8887, 135(2016), 35-38

## **Presenter Biography**



Kyung-Ho Kim received a B.S. degree from Sungkyunkwan University, South Korea, in 2021. He is currently studying for a Ph.D. degree in Mechanical engineering at Sungkyunkwan University. His research interests are path planning and control of autonomous vehicles and integrated chassis systems.



Hoyong Na received B.S. and M.S. degrees in mechanical engineering from Korea University of Technology and Education, Cheonan, South Korea, in 2017 and 2019, respectively. he is currently pursuing a Ph.D. degree with the School of Mechanical Engineering at Sungkyunkwan university, Suwon. His research interests include integrated chassis control systems, electrified power-train control systems, and motion control systems for future e-mobility solutions.



Jihyeok Ahn received a B.S. degree from Sungkyunkwan University, South Korea, in 2022. He is currently studying for an M.S. degree in Mechanical engineering at Sungkyunkwan University. His interests are path planning for autonomous vehicles and powertrain control integrated with AI.



Sujin Shin received a B.S. degree from Sookmyung Women's University, Korea, in 2022. She is currently studying for an M.S. degree in Mechanical Engineering at Sungkyunkwan University. Her interests are control of autonomous vehicles and vehicle simulation.



Sung-Ho Hwang received B.S., M.S., and Ph.D. degrees in mechanical design and production engineering from Seoul National University, Seoul, South Korea, in 1988, 1990, and 1997. From 1992 to 2002, he was a Senior Researcher with the Korea Institute of Industrial Technology, Cheonan. Since 2002, he has been a Professor at the School of Mechanical Engineering, Sungkyunkwan University, Suwon, South Korea. He authorizes two books, more than 100 articles, and more than 20 inventions. His research has focused on fundamental problems of dynamic systems, measurements, and controls in automotive applications, such as powertrains, electronically controlled chassis, and electric drive systems.